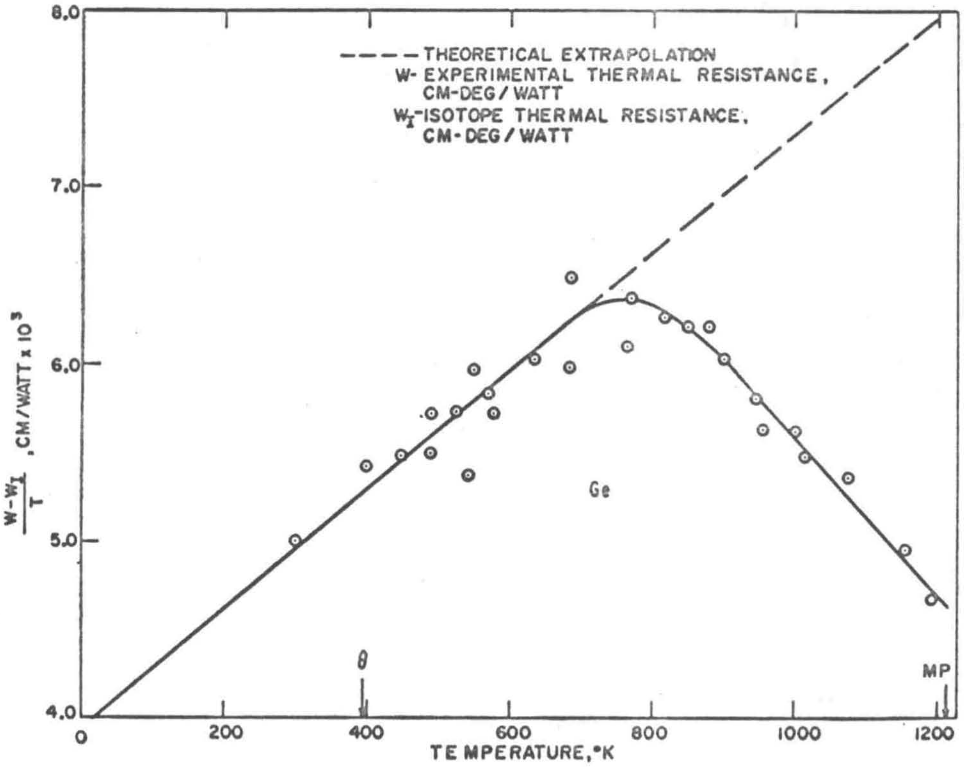


FIG. 11. The function $(W-W_I)T^{-1}$ versus $T$ for Ge as a means of determining the extrapolated lattice thermal resistivity at high temperatures.

Thus

$$K=K_g+K_e+K_r, \tag{7}$$

where $K_g$ is the phonon, $K_e$ is the electronic, and $K_r$ is the radiative or photon contribution to $K$.

Let us first consider the lattice thermal conductivity $K_g$. At high temperatures we have to consider the relaxation times $\tau_I$ and $\tau_U$ given in Eqs. (3) and (4), but we can neglect $\tau_B$ in Eq. (6). Equation (4) gives the relaxation time for 3-phonon umklapp processes. However, for $300°\text{K}<T<1681°\text{K}$, we are in the range of $T$ comparable to or greater than the Debye temperature $\theta$ for both Si and Ge. In this range, it may be necessary to consider the relaxation times for four-phonon processes, as Pomeranchuck[45–47] has pointed out. He gives a relaxation time for these higher order ($H$) processes as:

$$\tau_H^{-1}=B_H\omega^2T^2, \tag{8}$$

with $B_H$ a constant. $K_g$ can be evaluated for $T>\theta$ from Eqs. (1), (3), (4), and (8) if $\tau_C^{-1}$ is taken as

$$\tau_C^{-1}=\tau_U^{-1}+\tau_H^{-1}+\tau_I^{-1}.$$

In the region where $T>\theta$ the quantity $x$ in Eq. (1) is small, and the integral simplifies to

$$K=\frac{k}{2\pi^2v}\left(\frac{kT}{h}\right)^3\int_0^{\theta/T}\tau_C x^2dx. \tag{9}$$

Also the exponential factor in $B_U$ disappears to make $B_U$ temperature-independent. Thus

$$\tau_C^{-1}=(B_UT+B_HT^2)\omega^2+A\omega^4. \tag{10}$$

For $T\geq\theta$ the isotope scattering is much less important than the phonon-phonon scattering. In this limit Eqs. (9) and (10) can be reduced by the method used by Ambegaokar,[48] to

$$K_g^{-1}\equiv W_g=W_U+W_H+W_I,$$

where

$$\begin{aligned} W_U&=\pi vhB_UT/\theta k^2,\\ W_H&=\pi vhB_HT^2/\theta k^2,\\ W_I&=4\pi^2V_0\theta\Gamma/hv^2. \end{aligned} \tag{11}$$

This reduction requires $W_g\gg W_I$. This condition is fulfilled for Si and Ge at high temperatures. The only really unknown quantity in Eq. (11) is $B_H$. The quantities $B_U$ and $B_H$ can be evaluated experimentally from a plot of $(W_g-W_I)T^{-1}$ versus $T$. The quantity $W_I$ is, except for a difference of a factor of 12 in the definition of $\Gamma$, the same as that given by Ambegaokar.[48] For Si one obtains $W_I=0.033$ cm deg/W. For Ge the value is $W_I=0.17$ cm deg/W using $V_0=2.26\times10^{-23}$ cm³, $\theta=395°\text{K}$,[49] $\Gamma=4.90\times10^{-5}$,[50] and $v=3.94\times10^5$ cm/sec.

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